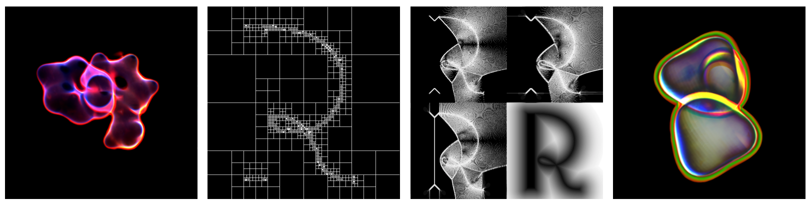
Adaptively Sampled Distance Fields (ADFs)

Representing Shape for Computer Graphics



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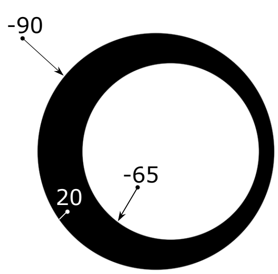
Outline

- Overview of ADFs
  - definition
  - advantages
  - instantiations
- Algorithms for octree-based ADFs
  - specifics of octree-based ADFs
  - generating, rendering, and triangulating ADFs
- Applications
  - sculpting, scanning, meshing, modeling, machining ...

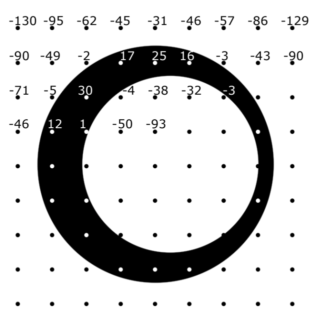
Distance Fields

- A distance field is a scalar field that
  - specifies the distance to a shape ...
  - where the distance may be signed to distinguish between the inside and outside of the shape
- Distance
  - can be defined very generally (e.g., non-Euclidean)
  - minimum Euclidean distance is used for most of this presentation (with the exception of the volumetric molecules)

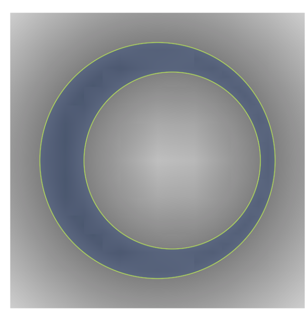
Distance Fields



2D shape with sampled distances to the surface



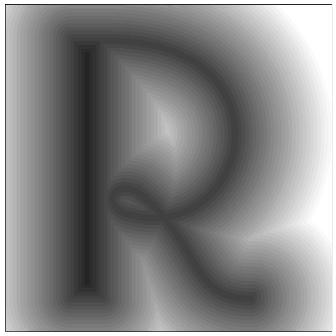
Regularly sampled distance values



2D distance field

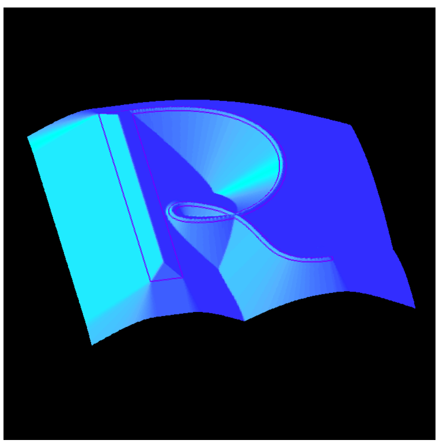
2D Distance Field

R shape



Distance field of R

2D Distance Field

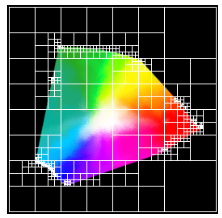


3D visualization of distance field of R

Shape

- By *shape* we mean more than just the 3D geometry of physical objects. Shape can have arbitrary dimension and be derived from simulated or measured data.

Color printer



Color gamut

Conceptual Advantages of Distance Fields

- Represent more than the surface
  - object interior and the space in which the object sits
- Gains in efficiency and quality because
  - distance fields vary “smoothly”
  - are defined throughout space
- Gradient of the distance field yields
  - surface normal for points on the surface
  - direction to closest surface point for points off the surface

Practical Advantages of Distance Fields

- Smooth surface reconstruction
  - continuous reconstruction of a smooth field
- Trivial inside/outside and proximity testing
  - using sign and magnitude of the distance field
- Fast and simple Boolean operations
  - intersection: $\text{dist}(A \cap B) = \min(\text{dist}(A), \text{dist}(B))$
  - union: $\text{dist}(A \cup B) = \max(\text{dist}(A), \text{dist}(B))$
- Fast and simple surface offsetting
  - offset by d: $\text{dist}(A_{\text{offset}}) = \text{dist}(A) + d$
- Enables geometric queries such as closest point
  - using gradient and magnitude of the distance field

Sampled Distance Fields

- Similar to sampled images, insufficient sampling of distance fields results in aliasing
- Because fine detail requires dense sampling, excessive memory is required with *regularly* sampled distance fields when *any* fine detail is present

Adaptively Sampled Distance Fields

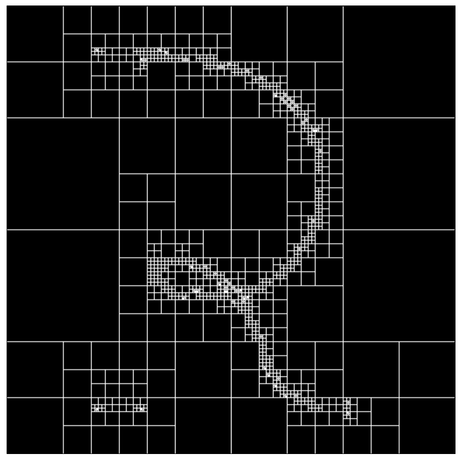
- Detail-directed sampling
  - high sampling rates only where needed
- Spatial data structure
  - fast localization for efficient processing
- ADFs consist of
  - adaptively sampled distance values …
  - organized in a spatial data structure …
  - with a method for reconstructing the distance field from the sampled distance values

ADF Instantiations

- Spatial data structures
  - octrees
  - wavelets
  - multi-resolution tetrahedral meshes …
- Reconstruction functions
  - trilinear interpolation
  - B-spline wavelet synthesis
  - barycentric interpolation ...

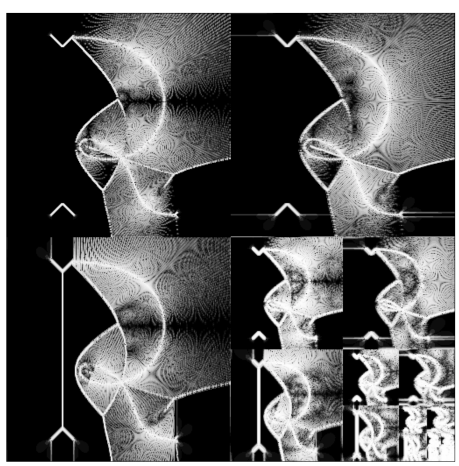
Quadtree

2D Spatial Data Structures – An Example



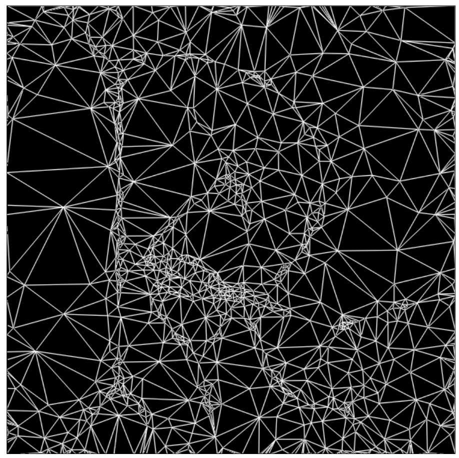
Wavelets

2D Spatial Data Structures – An Example



Multi-resolution Triangulation

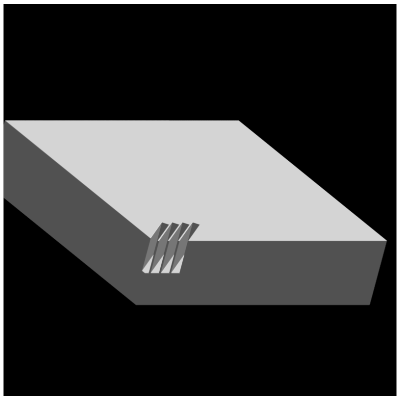
2D Spatial Data Structures – An Example

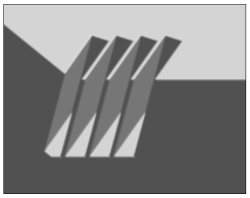


A Gallery of Examples – A Carved Vase

Illustrates smooth surface reconstruction, fine carving, and representation of algebraic complexity

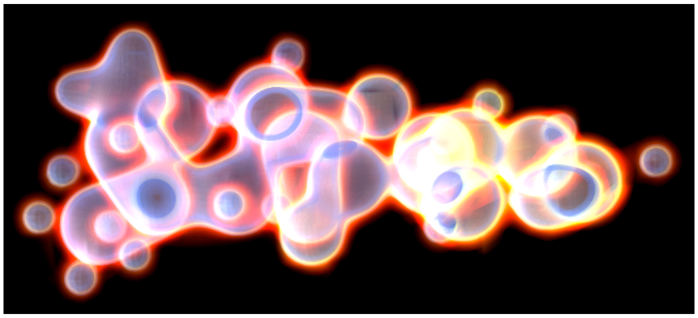
A Gallery of Examples – A Carved Slab





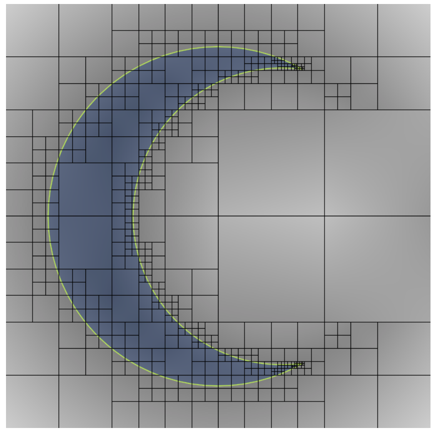
Illustrates sharp corners and precise cuts

A Gallery of Examples – A Volume Rendered Molecule



Illustrates volume rendering of ADFs, semi-transparency, thick surfaces, and distance-based turbulence

A Gallery of Examples – A 2D Crescent



ADFs provide:

- spatial hierarchy
- distance field
- object surface
- object interior
- object exterior
- surface normal (gradient at surface)
- direction to closest surface point (gradient off surface)

ADFs consolidate the data needed to represent complex objects

ADFs - A Unifying Representation

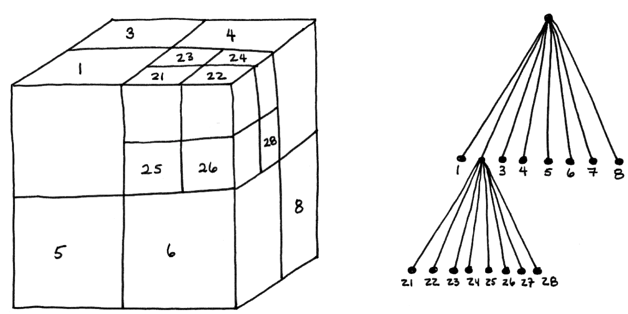
- Represent surfaces, volumes, and implicit functions
- Represent sharp edges, organic surfaces, thin-membranes, and semi-transparent substances
- Consolidate multiple structures for complex objects (e.g., for collision detection, LOD construction, and dynamic meshing)
- Can store auxiliary data in cells or at cell vertices (e.g., color and texture)

Algorithms for Octree-based ADFs

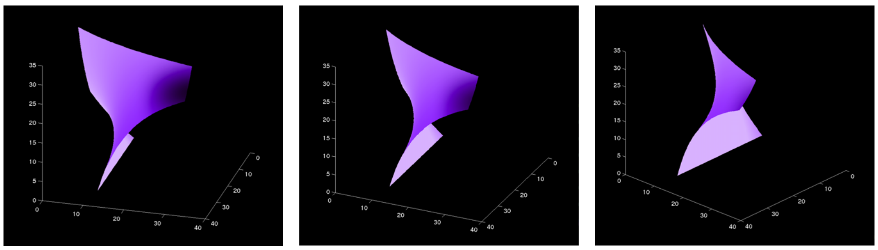
- Specifics of octree-based ADFs
- Generating ADFs
- Rendering ADFs
- Triangulating ADFs

Octree-based ADFs

- A distance value is stored for each cell corner in the octree
- Distances and gradients are estimated from the stored values using trilinear reconstruction

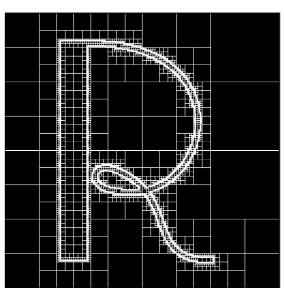


Reconstruction

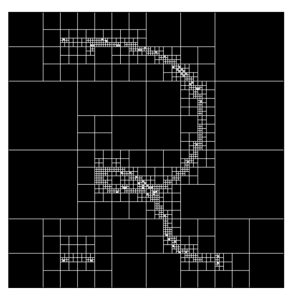


A single trilinear field can represent highly curved surfaces

Comparison of 3-color Quadtrees and ADFs

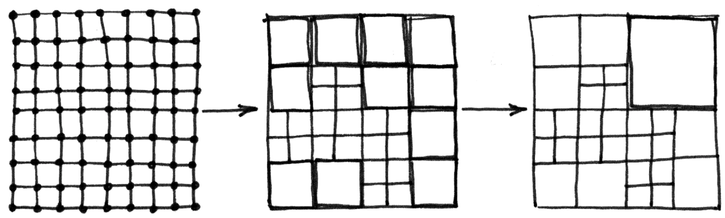


23,573 cells (3-color)



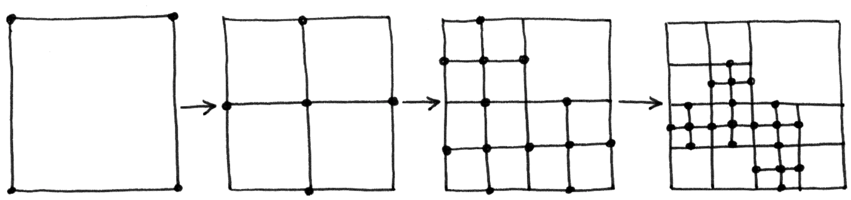
1713 cells (ADF)

Bottom-up Generation



Fully populate Recursively coalesce

Top-down Generation



Initialize root cell Recursively subdivide

Tiled Generation

- Reduced memory requirements
- Better memory coherency
- Reduced computation

```
tiledGeneration(genParams, distanceFunc)
  // cells: block of storage for cells
  // dists: block of storage for final distance values
  // tileVol: temporary volume for computed and
  // reconstructed distance values
  // bitFlagVol: volume of bit flags to indicate
  // validity of distance values in tileVol
  // cell: current candidate for tiled subdivision
  // tileDepth: L (requires (2^(L+1))^3 volume - the L+1
  // level is used to compute cell errors for level L)
  // maxADFLevel: preset max level of ADF (e.g., 12)

  maxLevel = tileDepth
  cell = getNextCell(cells)
  initializeCell(cell, NULL) (i.e., root cell)

  while (cell)
    setAllBitFlagVolInvalid(bitFlagVol)
    if (cell.level == maxLevel)
      maxLevel = min(maxADFLevel, maxLevel + tileDepth)
    recurSubdivToMaxLevel(cell,maxLevel,maxADFLevel)
    addValidDistsToDistsArray(tileVol, dists)
    cell = getNextCandidateForSubdiv(cells)
```

```
initializeCell(cell, parent)
  initCellFields(cell, parent, bbox, level)
  for (error = 0, pt = cell, face, and edge centers)
    if (isBitFlagVolValidAtPt(pt))
      comp = getTileComputedDistAtPt(pt)
      recon = getTileReconstructedDistAtPt(pt)
    else
      comp = computeDistAtPt(pt)
      recon = reconstructDistAtPt(cell, pt)
      setBitFlagVolValidAtPt(pt)
    error = max(error, abs(comp - recon))
  setCellError(error)
```

```
recurSubdivToMaxLevel(cell, maxLevel, maxADFLevel)
  // Trivially exclude INTERIOR and EXTERIOR cells
  // from further subdivision
  pt = getCellCenter(cell)
  if (abs(getTileComputedDistAtPt(pt)) >
      getCellHalfDiagonal(cell))
    // cell.type is INTERIOR or EXTERIOR
    setCellTypeFromCellDistValues(cell)
    return

  // Stop subdividing when error criterion is met
  if (cell.error < maxError)
    // cell.type is INTERIOR, EXTERIOR, or BOUNDARY
    setCellTypeFromCellDistValues(cell)
    return

  // Stop subdividing when maxLevel is reached
  if (cell.level >= maxLevel)
    // cell.type is INTERIOR, EXTERIOR, or BOUNDARY
    setCellTypeFromCellDistValues(cell)
    if (cell.level < maxADFLevel)
      // Tag cell as candidate for next layer
      setCandidateForSubdiv(cell)
    return

  // Recursively subdivide all children
  for (each of the cell's 8 children)
    child = getNextCell(cells)
    initializeCell(child, cell)
    recurSubdivToMaxLevel(child, maxLevel, maxADFLevel)

  // cell.type is INTERIOR, EXTERIOR, or BOUNDARY
  setCellTypeFromChildrenCellTypes(cell)

  // Coalesce INTERIOR and EXTERIOR cells
  if (cell.type != BOUNDARY) coalesceCell(cell)
```

Tiled Generation Pseudocode

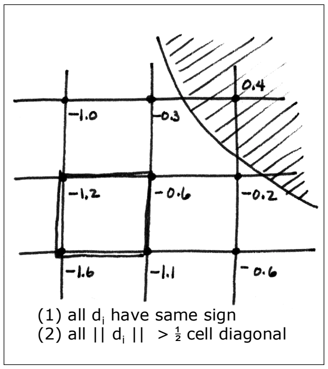
Tiled Generation – Overview

- Recursively subdivide root cell to a level *L*
- Cells at level *L* requiring further subdivision are appended to a list of candidate cells, *C-list*
- These candidate cells are recursively subdivided between levels *L* and *2L*, where new candidate cells are produced and appended to *C-list*
- Repeat layered production of candidate cells (*2L* to *3L*, etc.) until *C-list* is empty

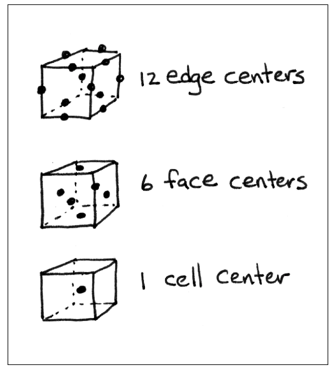
Tiled Generation – Candidate Cells

- A cell becomes a candidate for further subdivision when all of the following are true:
  - it is a leaf cell of level *L*, or *2L*, or *3L*, etc.
  - it can not be trivially determined to be an interior or exterior cell
  - it does not satisfy a specified error criterion
  - its level is below a specified maximum ADF level

Tests for Candidate Cells



Test to trivially determine if a cell is interior or exterior



19 test points to determine cell error

Tiled Generation – Tiling

- For each candidate cell, computed and reconstructed distances are produced *only* as needed during subdivision
- These distances are stored in a ***tile***, a regularly sampled volume
- The tile resides in cache memory and its size determines *L*
- A volume of bit flags keeps track of valid distances in the tile to ensure that distances are computed only once

Tiled Generation – Tiling

- For coherency, cells and final distances are stored in two separate contiguous memory blocks
- After a candidate cell has been processed, valid distances in the tile are appended to the block of final distances
- Special care is taken at tile boundaries to ensure that distances are *never* duplicated for neighboring cells

Tiled Generation – Cache Efficiency

- Tile sizes can be tuned to the CPU cache architecture
- For current Pentium systems, a tile size of 16^3 has worked most effectively
- Using a separate bit flag volume further enhances cache effectiveness and provides fast invalidation of tile distances prior to processing each candidate cell

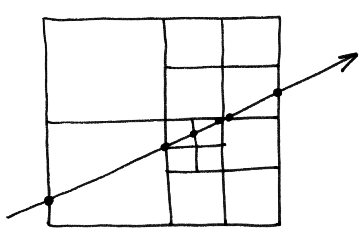
Rendering

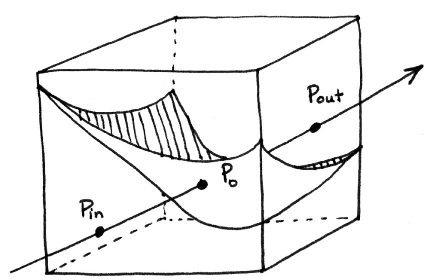
- Ray casting
- Adaptive ray casting
- Point-based rendering
- Triangles

Ray Casting

Ray-surface Intersection with a Cubic Solver

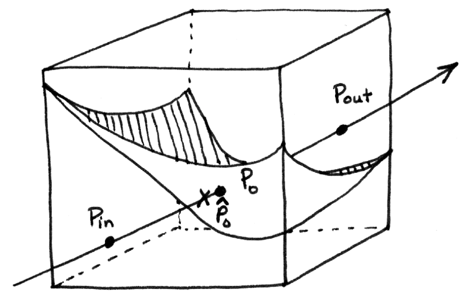
- See Parker *et al.*, ”Interactive Ray Tracing for Volume Visualization”





Ray Casting
Ray-surface Intersection with a Linear Solver

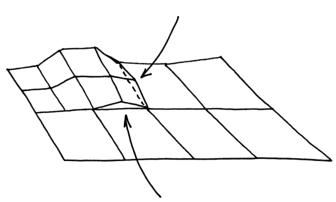
- Assume that distances vary linearly along the ray
- Determine the zero-crossing within the cell given distances at the points where the ray enters and exits the cell

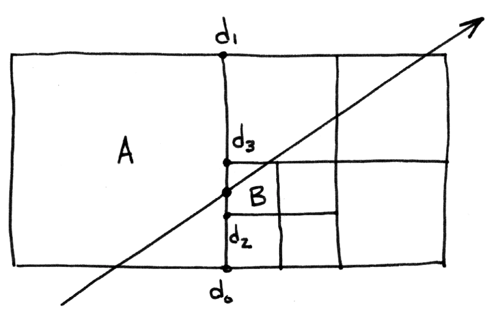


$$\hat{P}_o = P_{in} \cdot (1-t) + P_{out} \cdot t$$

Ray Casting
Crackless Surface Rendering with the Linear Solver

- Set the distance at the entry point of a cell equal to the distance computed for the exit point of the previous cell

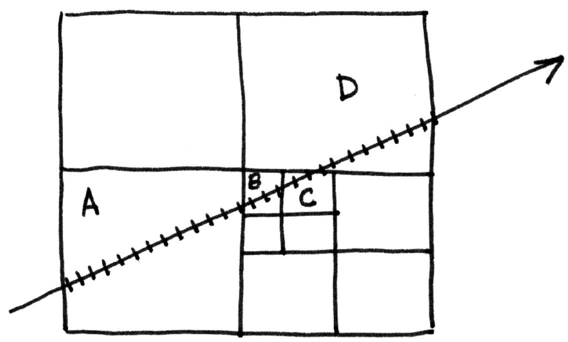




$$d_{out}(A) \stackrel{?}{=} d_{in}(B)$$

Ray Casting
Volume Rendering

- Colors and opacities are accumulated at equally spaced samples along each ray



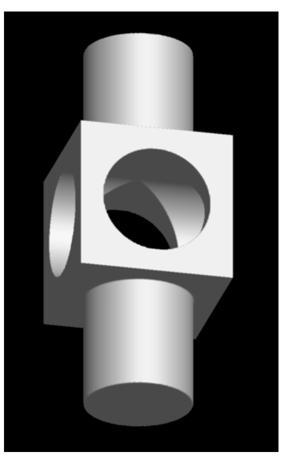
Adaptive Ray Casting

- The image region to be rendered is divided into a hierarchy of image tiles
- The subdivision of each tile is guided by a perceptually-based predicate
- Pixels within image tiles of size greater than 1x1 are bilinearly interpolated to produce the image
- Rays are cast into the ADF at tile corners and intersected with the surface using the linear solver

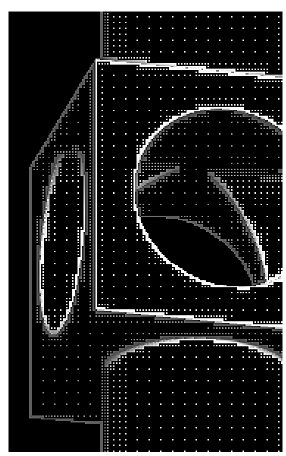
Adaptive Ray Casting

- The predicate individually weights the contrast in the red, green, and blue channels and the variance in depth-from-camera across the tile
  - See Mitchell , SIGGRAPH'87, and Bolin and Meyer, SIGGRAPH'98
- Results in a typical 6:1 reduction in rendering time over non-adaptive ray casting

Adaptive Ray Casting



Adaptively ray cast ADF



Rays cast to render part of the left image

Point-based Rendering

- Determine the number of points to generate in each boundary leaf cell
  - Compute an estimate of the object's surface area within each boundary leaf cell *areaCell* and the total estimated surface area of the object,
    areaObject = Σ *areaCell*
  - Set the number of points in each cell *nPtsCell* proportional to *areaCell / areaObject*

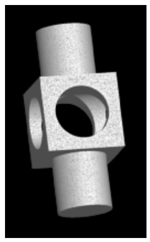
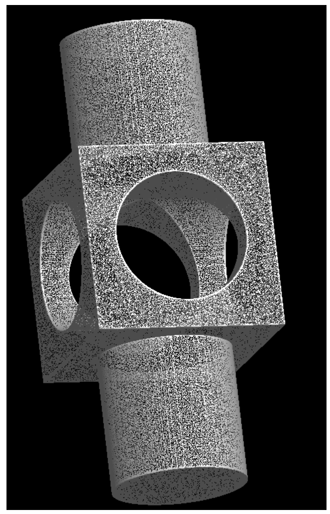
- For each boundary leaf cell in the ADF
  - Generate *nPtsCell* random points in the cell
  - Move each point to the object's surface using the distance and gradient at the point

Point-based Rendering

Pseudocode

```
generatePoints(adf, points, nPts, maxPtsToGen)
  // Estimate object's surface area within each boundary leaf
  // cell and the total object's surface area
  for (areaObject = 0, level = 0 to maxADFLevel)
    nCellsAtLevel = getNumBoundaryLeafCellsAtLevel(adf, level)
    areaCell[level] = sqr(cellSize(level))
    areaObject += nCellsAtLevel * areaCell[level]

  // nPtsCell is proportional to areaCell / areaObject
  for (level = 0 to maxADFLevel)
    nPtsAtLevel[level] = maxPtsToGen * areaCell[level] / areaObject

  // For each boundary leaf cell, generate cell points
  // and move each point to the surface
  for (nPts = 0, cell = each boundary leaf cell of adf)
    nPtsCell = nPtsAtLevel[cell.level]
    while (nPtsCell--)
      pt = generateRandomPositionInCell(cell)
      d = reconstructDistAtPt(cell, pt)
      n = reconstructNormalizedGradtAtPt(cell, pt)
      pt += d * n
      n = reconstructNormalizedGradtAtPt(cell, pt)
      setPointAttributes(pt, n, points, nPts++)
```

Point-based Rendering

An ADF rendered as points at two different scales

Triangle Rendering

- ADFs can also be rendered by triangulating the surface and using graphics hardware to rasterize the triangles
- Triangulation is fast
  - 200,000 triangles in 0.37 seconds, Pentium IV
  - 2,000 triangles in < 0.01 seconds
- The triangulation produces models that are orientable and closed

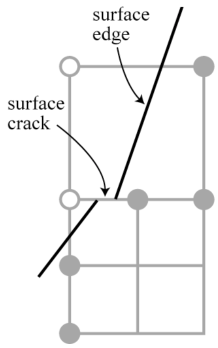
Triangulation

- ***Seed*** – Each boundary leaf cell of the ADF is assigned a vertex that is initially placed at the cell's center
- ***Join*** – Vertices of neighboring cells are joined to form triangles
- ***Relax*** – Vertices are moved to the surface using the distance field
- ***Improve*** – Vertices are moved over the surface towards their average neighbors' position to improve triangle quality

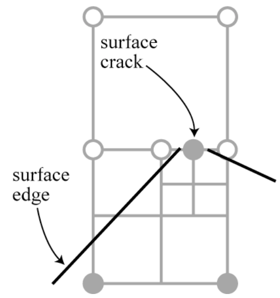
Triangulation

- Vertices are ***joined*** to form triangles using the following observations
  - A triangle joins the vertices of 3 neighboring cells that share a common edge (hence triangles are associated with cell edges)
  - A triangle is associated with an edge only if that edge has a zero crossing of the distance field
  - The orientation of the triangle can be derived from the orientation of the edge it crosses
  - In order to avoid making redundant triangles, we consider 6 of the 12 possible edges for each cell

Triangulation – Surface Cracks



Most triangulation algorithms for adaptive grids suffer from this type of crack; our algorithm ***does not***



As with other algorithms, this type of crack occurs ***very rarely*** but we can prevent it with a simple pre-conditioning step

Triangulation – Pre-conditioning

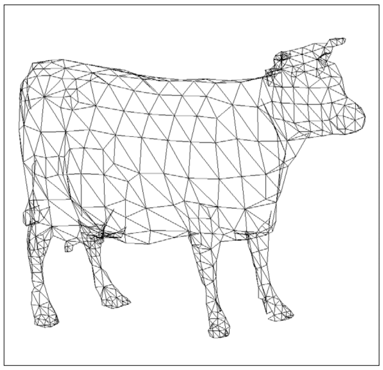
- In 3D, the pre-conditioning step compares the number of zero-crossings of the iso-surface for each face of each boundary leaf cell to the total number of zero-crossings for faces of the cell's face-adjacent neighbors that are shared with the cell
- When the number of zero-crossings are not equal for any face, the cell is subdivided using distance values from its face-adjacent neighbors until the number of zero-crossings match

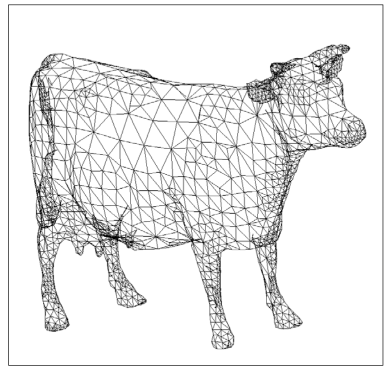
```
triangulateADF(adf)
  // vertices: storage for vertices
  // triangles: storage for triangles

  // Initialize triangles vertices at cell centers
  // and associate each vertex with its cell

  for (cell = each boundary leaf cell of adf)
    v = getNextVertex(vertices)
    associateVertexWithCell(cell, v)
    v.position = getCellCenter(cell)

  // Make triangles. Each cell edge joins two cell
  // faces face1 and face2 which are ordered to ensure
  // a consistent triangle orientation (see EdgeFace
  // table below). For a given cell edge and face,
  // getFaceNeighborVertex returns either the vertex of
  // the cell's face-adjacent neighbor if the
  // face-adjacent neighbor is the same size or larger
  // than the cell, OR, the vertex of the unique child
  // cell (uniqueness is guaranteed by a
  // pre-conditioning step) of the face-adjacent
  // neighbor that is both adjacent to the face and
  // has a zero-crossing on the edge

  for (cell = each boundary leaf cell of adf)
    for (edge = cell's up-right, down-left, up-front,
    down-back, front-right, and back-left edges)
      if (surfaceCrossesEdge(edge))
        face1 = EdgeFace[edge].face1
        face2 = EdgeFace[edge].face2
        v0 = getCellsAssociatedVertex(cell)
        v1 = getFaceNeighborVertex(face1, edge)
        v2 = getFaceNeighborVertex(face2, edge)
        t = getNextTriangle(triangles)
        if (edgeOrientation(edge) > 0)
          t.v0 = v0, t.v1 = v1, t.v2 = v2
        else
          t.v0 = v0, t.v1 = v2, t.v2 = v1

  // Relax each vertex to the surface and then along
  // the tangent plane at the relaxed position towards
  // the average neighbor position

  for (each vertex)
    v = getVertexPosition(vertex)
    u = getAveragePositionOfNeighborVertices(vertex)
    cell = getVertexCell(vertex)
    d = reconstructDistAtPt(cell, v)
    n = reconstructNormalizedGradtAtPt(cell, v)
    v += d * n
    v += (u - v) - n · (u - v)


// -----------------------------------------
// EdgeFace table:
//   edge              face1       face2
//   ----------        -----       -----
//   up-right          up          right
//   down-left         down        left
//   up-front          up          front
//   down-back         down        back
//   front-right       front       right
//   back-left         back        left
```

Triangulation – Level-of-Detail

- The octree is traversed and vertices are ***seeded*** into boundary cells whose maximum error satisfies a user-specified threshold
- Cells below these cells in the hierarchy are ignored
- The error threshold can be varied continuously enabling fine control over the number of triangles generated
- Time to produce an LOD model is proportional to the number of vertices in the output mesh

Triangulation – Level-of-Detail



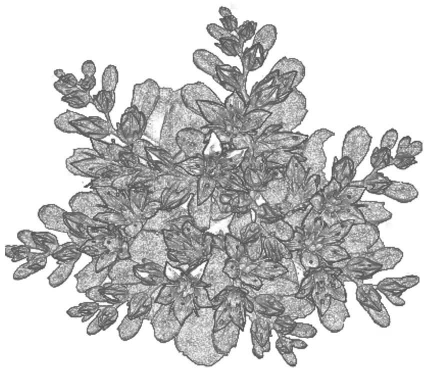


Applications

- Sculpting
- 3D scanning
- Dynamic meshing
- Physically-based modeling
- Color management
- Volumetric effects
- Machining

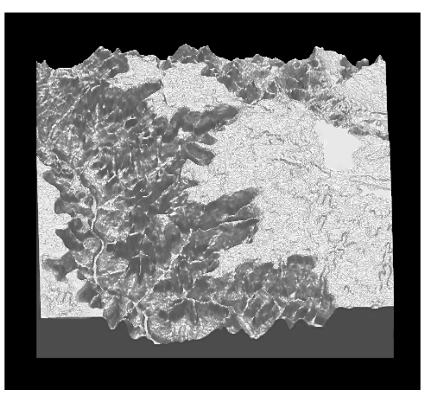
Sculpting

“Kizamu: A System for Sculpting Digital Characters”



- ADFs can represent both smooth surfaces and sharp corners without excessive memory
- Carving is direct, intuitive, and fast
- Does not require control point manipulation or trimming
- The distance field can be used to position and orient the sculpting tool or to constrain carving

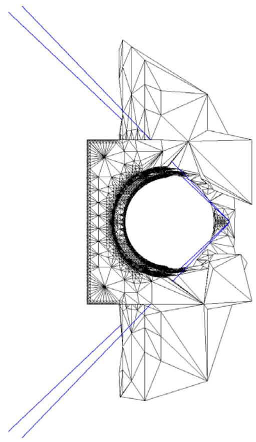
3D Scanning



- Use of distance fields provides more robust, water-tight surfaces
- ADFs result in significant savings in memory and distance computations
- Resultant models can be directly sculpted to correct the scanned data
- Fast new triangulation method produces optimal triangle meshes from the ADF

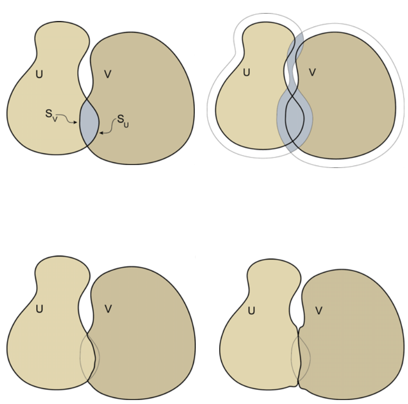
Dynamic Meshing

Level-of-Detail and View Dependent Triangulation



- ADF octree provides hierarchical structure for generating LOD models
- View-dependent meshing uses ADF hierarchy, cell size, and cell gradients
- ADF cell error enables fine control over triangle count in LOD meshes
- Real-time ADF triangulation algorithm produces meshes that are orientable and closed

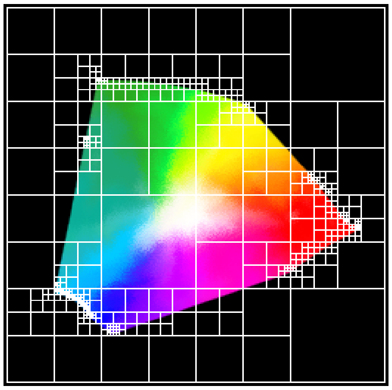
Physically-based Modeling



- ADFs provide a compact representation of complex surfaces
- ADF spatial hierarchy and trivial inside/outside tests enable fast collision detection
- Distance field provides penetration depths for computing impact forces
- Distance field allows computation of material-dependent contact deformation

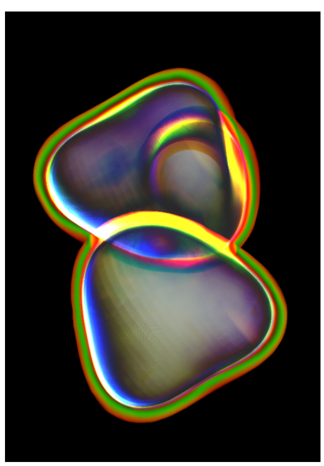
Color Management

Representing Color Gamuts



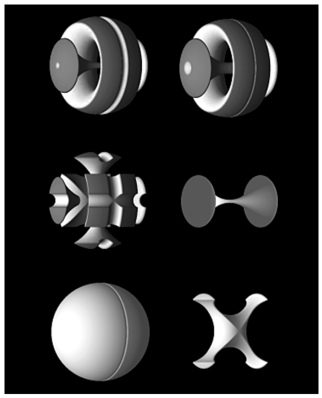
- ADF distance field enables a fast, simple out-of-gamut test
- ADFs provide a compact representation of complex gamut shapes
- Gamut test is very accurate near the gamut surface
- Distance and gradient indicate how far out of gamut a color lies and the direction to the nearest in-gamut color

Volumetric Effects



- Offset surfaces can be used to render thick, translucent surfaces
- Volume texture can be added within the thick surface
- Distance values away from the surface can be used for special effects (e.g., turbulent haze)
- Octree and distance field allow space-leaping and other methods to speed up volume rendering

Machining



- ADFs represent surfaces, object interiors, and the material to be removed
- ADFs represent smooth surfaces and very fine detail
- Trivial inside/outside and proximity tests are useful for designing tool paths
- Gradients can be used to select tool orientation
- Offset surfaces can be used for rough cutting in coarse-to-fine milling

For More Information
At Siggraph 2001

- Paper presentation:
  - *"Kizamu: A System for Sculpting Digital Characters",* Wednesday, 15 August, 10:30 am
- Sketches:
  - *"Dynamic Meshing Using Adaptively Sampled Distance Fields",* Wednesday, 15 August, 4:30 pm
  - *"A Computationally Efficient Framework for Modeling Soft Body Impact",* Thursday, 18 August, 8:30 am
  - *"Computing 3D Geometry Directly from Range Images",* Friday, 17 August, 2:20 pm

For More Information
In Your Course Notes

- A nearly final version of the Kizamu paper, SIGGRAPH 2001 and MERL Technical Report TR2001-08
- *"A New Representation for Device Color Gamuts",* MERL Technical Report TR2001-09
- *"Computing 3D Geometry Directly from Range Images",* SIGGRAPH 2001 Technical Sketch and MERL Technical Report TR2001-10
- *"A Computationally Efficient Framework for Modeling Soft Body Impact",* SIGGRAPH 2001 Technical Sketch and MERL Technical Report TR2001-11
- *"A New Framework For Non-Photorealistic Rendering",* MERL Technical Report TR2001-12
- *"Dynamic Meshing Using Adaptively Sampled Distance Fields",* SIGGRAPH 2001 Technical Sketch and MERL Technical Report TR2001-13
- *"Adaptively Sampled Distance Fields: A General Representation of Shape for Computer Graphics",* SIGGRAPH 2000 and MERL Technical Report TR2000-15
- *"Using Distance Maps for Accurate Surface Representation in Sampled Volumes",* IEEE VolVis Symp. 1998 and MERL Technical Report TR99-25

The End