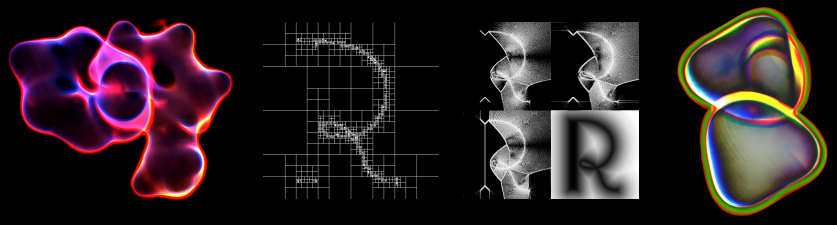
Adaptively Sampled Distance Fields:

A General Representation of Shape for Computer Graphics



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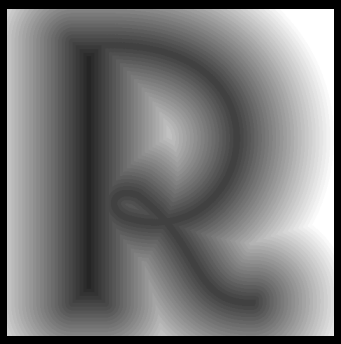
Distance Fields

- A distance field is a scalar field that
  - specifies the minimum distance to a shape ...
  - where the distance may be signed to distinguish between the inside and outside of the shape
- Distance is not restricted to Euclidean

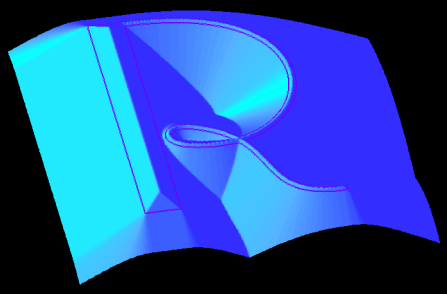
2D Euclidean Distance Field Example

R shape



Distance field of R

2D Euclidean Distance Field Example

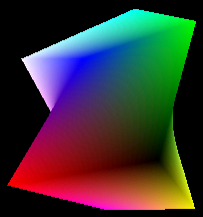


3D visualization of distance field of R

Shape

- By *shape* we mean more than just the 3D geometry of physical objects. Shape can have arbitrary dimension and be derived from simulated or measured data.

Color printer



Color gamut

Advantages for Shape Representation

- Represent more than the surface
  - object interior and the space in which the shape sits
- Gains in efficiency and quality because
  - distance fields vary "smoothly"
  - are defined throughout space

- Gradient of the distance field yields
  - surface normal for points on the surface
  - direction to closest surface point for points off the surface

Advantages for Shape Representation

- Smooth surface reconstruction
- Inside/outside and proximity testing
- Boolean operations
- Surface offsetting
- Geometric queries such as closest point
- Numerous applications
  - blending and filleting
  - morphing
  - rough cutting
  - collision detection
  - path planning

Sampled Distance Fields

- Similar to sampled images, insufficient sampling of distance fields results in aliasing
- Because fine detail requires dense sampling, excessive memory is required with *regularly* sampled distance fields when *any* fine detail is present

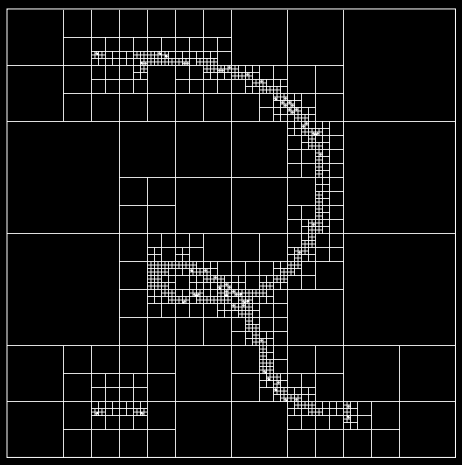
Adaptively Sampled Distance Fields (ADFs)

- Detail-directed sampling
  - high sampling rates only where needed
- Spatial data structure
  - fast localization for efficient processing
- ADFs consist of
  - adaptively sampled distance values …
  - organized in a spatial data structure …
  - with a method for reconstructing the distance field from sampled distance values

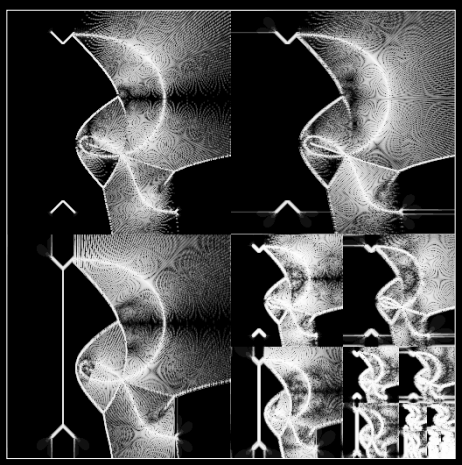
Various ADF Representations

- Spatial data structures
  - octrees
  - wavelets
  - multi-resolution tetrahedral meshes ...
- Reconstruction functions
  - trilinear
  - B-spline wavelet synthesis
  - barycentric ...

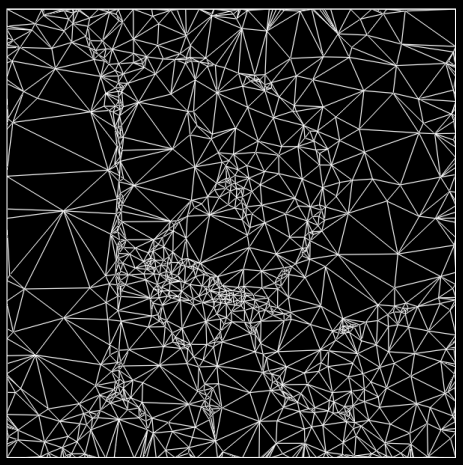
Examples of 2D Spatial Data Structures
Quadtree



Examples of 2D Spatial Data Structures
Wavelet



Examples of 2D Spatial Data Structures
Multi-resolution Triangulation



Related Work

Distance fields
Barerentzen, Sramek and Christensen, 2000
Breen, Mauch and Whitaker, 1998
Cohen-Or, Levin and Solomovici, 1997
Curlass and Levoy, 1996
Gibson, 1998
Kimmel, Kiryati and Bruckstein, 1998
Lengyel and Reichert, 1990
Payne and Toga, 1992
Schroeder, Lorensen and Linthicum, 1994
Yagel, Lu, Rubello and Miller, 1995
Zuiderveld, Koning and Viergever, 1992

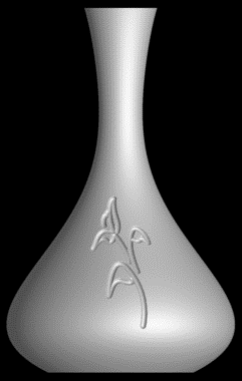
Volume sculpting
Avila and Sobierajski, 1996
Baerentzen, 1998
Galyean and Hughes, 1991
Wang and Kaufman, 1994

Implicit surfaces
Bloomenthal, 1997
Desbrun and Gascuel, 1995
Larcombe, 1994
Gascuel, 1998
Ricci, 1973

Multi-resolution volumes
Cignoni, De Floriani, Montani, Puppo and Scopigno, 1994
Hamann and Cehn, 1994
Ertl, Westerman and Grosso, 1998
Westermann, Sommer and Ertl, 1999

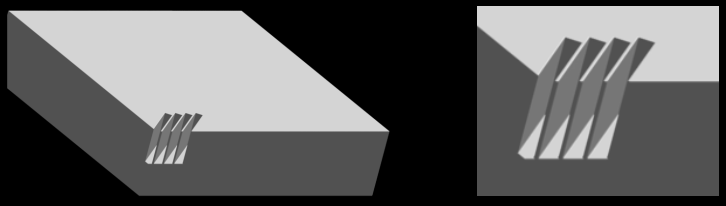
Level sets
Osher and Sethian, 1988
Sethian, 1996
Whitaker and Breen, 1998

A Gallery of Examples - A Carved Vase



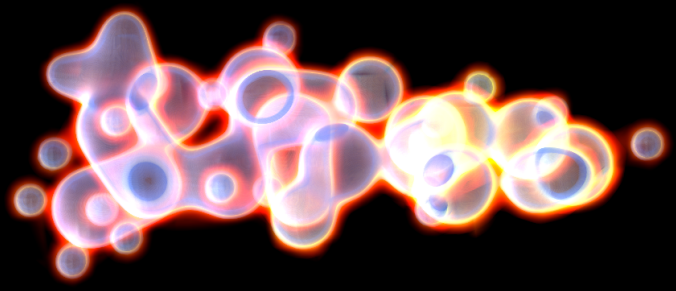
Illustrates smooth surface reconstruction, fine carving, and representation of algebraic complexity

A Gallery of Examples - A Carved Slab



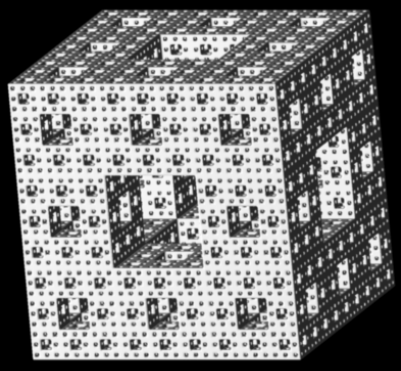
Illustrates sharp corners and precise cuts

A Gallery of Examples - A Volume Rendered Molecule



Illustrates volume rendering of ADFs, semi-transparency, thick surfaces, and distance-based turbulence

A Gallery of Examples - The Menger Sponge



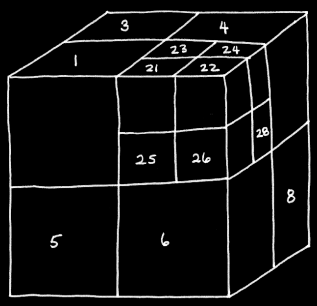
ADFs simplify the data structures needed to represent complex objects

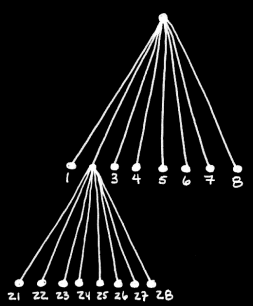
ADFs - A Unifying Data Stucture

- Represents surfaces, volumes and implicit functions
- Represents sharp edges, organic surfaces, thin-membranes and semi-transparent substances
- Consolidates multiple structures for complex objects
- Can store auxiliary data in cells or at cell vertices

An Example - Octree-based ADFs

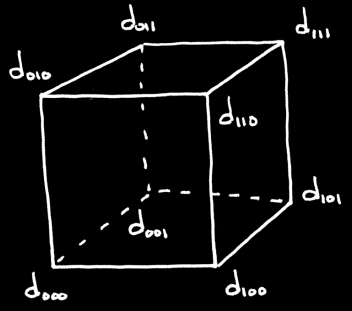
- Store distance values at cell vertices of an octree



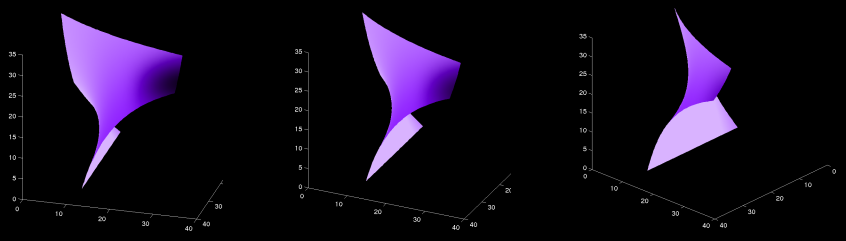


Reconstruction

- Distances and gradients are estimated using trilinear reconstruction

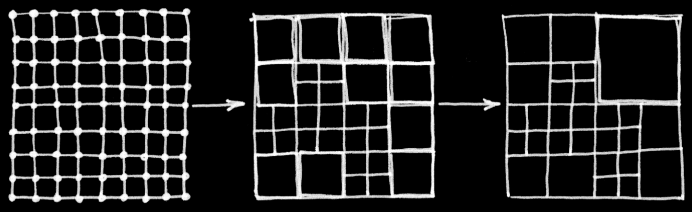


Reconstruction



A single trilinear field can represent highly curved surfaces

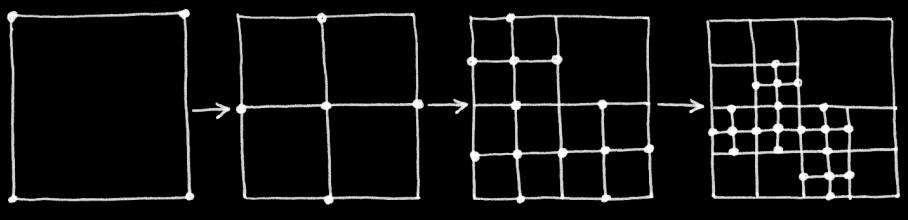
Bottom-up Generation



Fully populate

Recursively coalesce

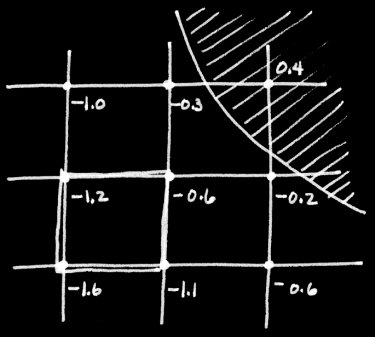
Top-down Generation



Initialize root cell

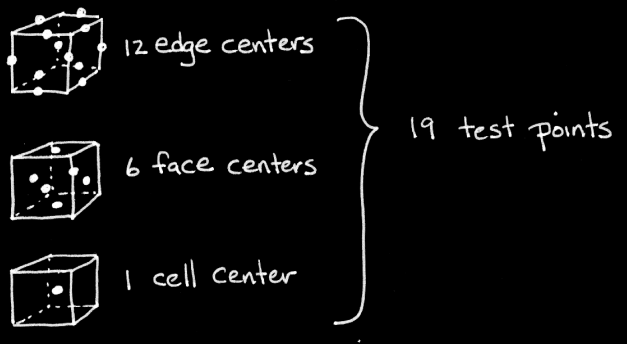
Recursively subdivide

Surface Exclusion

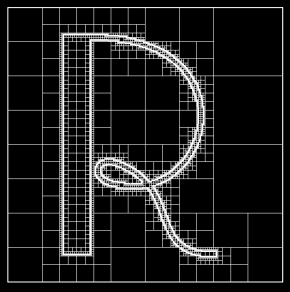


(1) all d_i have same sign
(2) all $|| d_i ||$ > ½ cell diagonal

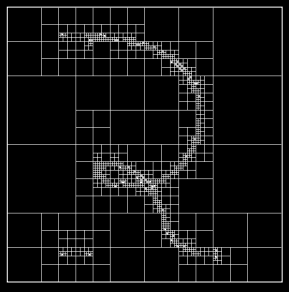
Predicate Test for Subdividing and Coalescing Cells



Comparison of 3-color Quadtrees and ADFs

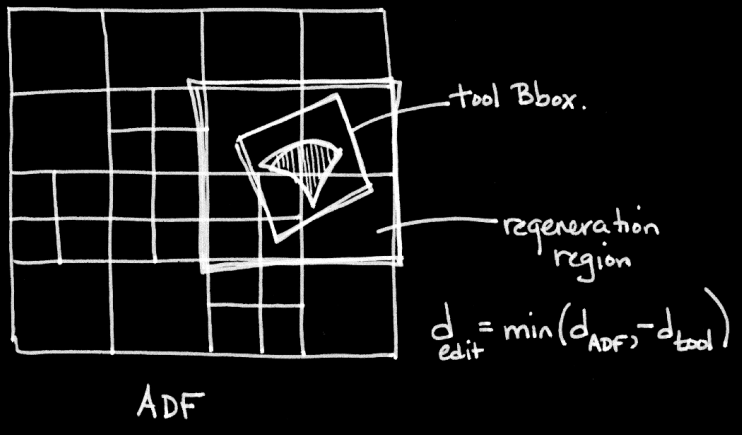


23,573 cells (3-color)

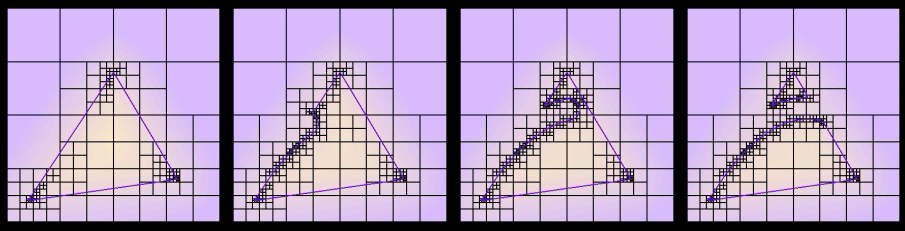


1713 cells (ADF)

Editing with Boolean Operations - Local ADF Regeneration



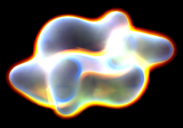
2D Editing with Boolean Difference Operator



Rendering

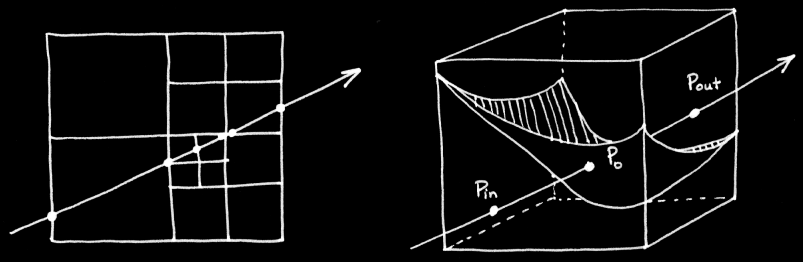
- Surface rendering
  - ray casting with analytic surface intersection

- Volume rendering
  - back-to-front sampled ray casting

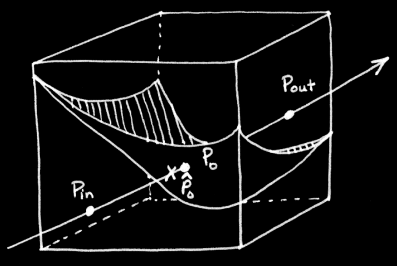


Surface Rendering - Cubic Solver

- See Parker, et al.

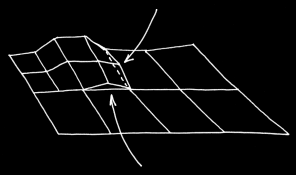


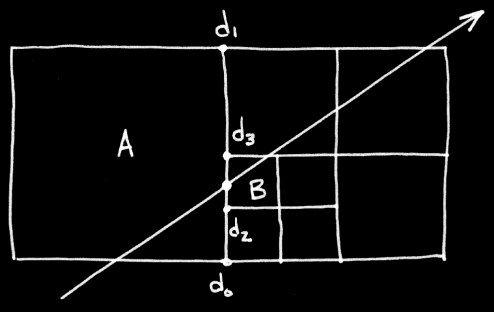
Surface Rendering - Linear Solver



$$\hat{P}_o = P_{in} \cdot (1-t) + P_{out} \cdot t$$

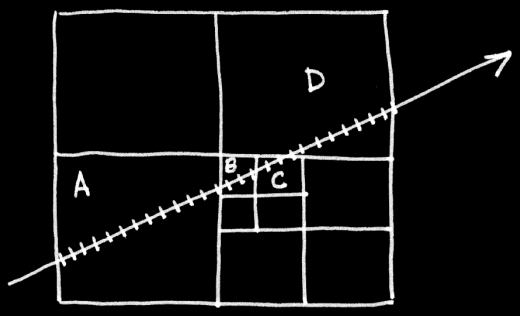
Crackless Surface Rendering with the Linear Solver





$d_{out}(A) \stackrel{?}{=} d_{in}(B)$

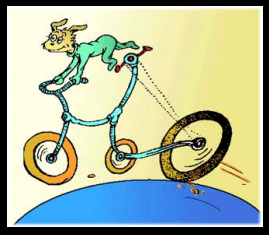
Volume Rendering



Applications

"It's a shirt. It's a sock. It's a glove. It's a hat. But it has other uses. Yes, far beyond that. You can use it for carpets. For pillows! For sheets! Or curtains! Or covers for bicycle seats!"

— The Lorax, Dr. Seuss



Applications - Carving

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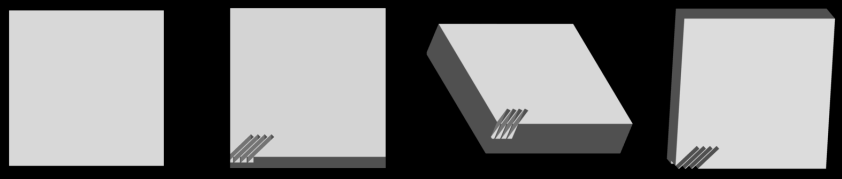
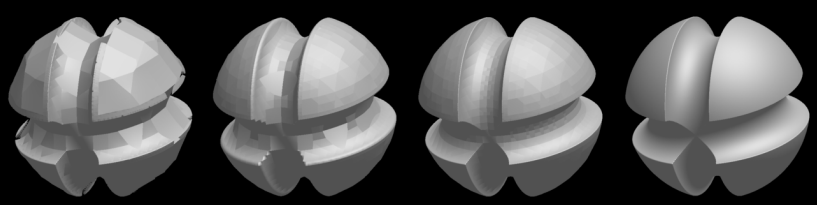


Applications - Level of Detail

- Different ways to compute LOD from an ADF
  - octree level cut-off
  - stored error cut-off



Other Application Areas

- Collision detection
- Color gamut representation
- Milling
- Path planning
- Volumetric effects

Conclusions

- ADFs are a unifying representation of shape
- They integrate numerous concepts in computer graphics including
  - the representation of geometry and volume data
  - a broad range of processing operations such as rendering, carving, LOD management, surface offsetting, and proximity testing